Statistics

Lecture 8

Feb 19-8:47 AM

Some Review

$5+8+7=20$ balls

A box has 5 Red, 8 white, and 7 Blue balls.

If we randomly select $\boxed{\text{one ball}}$, find

1) $P(\text{Red}) = \frac{5}{20} = \boxed{\frac{1}{4}}$

2) $P(\overline{\text{white}}) = 1 - P(\text{white})$
$= 1 - \frac{8}{20} = \frac{12}{20} = \boxed{\frac{3}{5}}$

3) $P(\underbrace{\text{Red and white}}_{\text{Impossible event}}) = \boxed{0}$

4) $P(\text{Red or white})$
$= \frac{5+8}{20} = \boxed{\frac{13}{20}}$

Jan 24-4:31 PM

Suppose $P(A) = .025$

1) write $P(A)$ in reduced fraction.

$.025$ $\boxed{\text{MATH}}$ $\boxed{1:\blacktriangleright\text{Frac}}$ $\boxed{\text{Enter}}$ $\frac{1}{40}$

2) find $P(\bar{A})$ in decimals.

$P(\bar{A}) = 1 - P(A) = 1 - .025 = \boxed{.975}$

3) Simplify $\frac{P(A)}{P(\bar{A})}$, express your answer as a ratio using : notation

$\frac{.025}{.975} = \frac{1}{39} \longrightarrow 1:39$

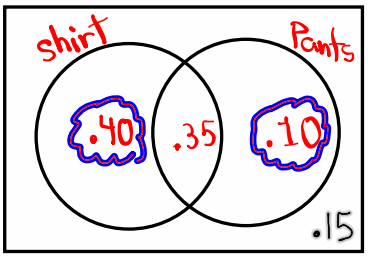
$.025$ $\boxed{\div}$ $.975$ $\boxed{\text{MATH}}$ $\boxed{1:\blacktriangleright\text{Frac}}$ $\boxed{\text{Enter}}$

Jan 24-4:36 PM

$P(\text{shirt}) = .75$

$P(\text{pants}) = .45$

$P(\underbrace{\text{shirt and Pants}}_{\text{overlap}}) = .35$



Total = 1

$P(\text{shirt only}) = .75 - .35 = .4$

$P(\text{Pants only}) = .45 - .35 = .1$

$P(\text{Shirts or Pants}) = P(\text{Shirts}) + P(\text{pants}) - P(S \,\&\, P)$

$= .75 + .45 - .35 = \boxed{.85}$

$P(\text{buy one, not both}) = .40 + .10$

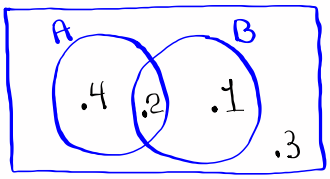
$= \boxed{.5}$

Jan 24-4:42 PM

De Morgan's Law:

$$\boxed{\begin{aligned} P(\bar{A} \text{ and } \bar{B}) &= P(\overline{A \text{ or } B}) \\ P(\bar{A} \text{ or } \bar{B}) &= P(\overline{A \text{ and } B}) \end{aligned}}$$

$P(A) = .6$, $P(B) = .3$, $P(A \text{ and } B) = .2$



Total = 1

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$= .6 + .3 - .2 = \boxed{.7}$

$P(\bar{A} \text{ and } \bar{B}) = P(\overline{A \text{ or } B}) = 1 - P(A \text{ or } B) = 1 - .7 = \boxed{.3}$

$P(\bar{A} \text{ or } \bar{B}) = P(\overline{A \text{ and } B}) = 1 - P(A \text{ and } B) = 1 - .2 = \boxed{.8}$

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Given: $P(A) = .65$, $P(B) = .75$, $P(A \text{ or } B) = .85$

1) $P(\bar{A}) = 1 - P(A) = \boxed{.35}$

2) $P(\overline{A \text{ or } B}) = 1 - P(A \text{ or } B) = \boxed{.15}$

3) $P(A \text{ and } B)$

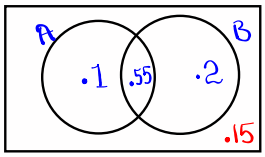
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$.85 = .65 + .75 - P(A \text{ and } B)$

$P(A \text{ and } B) = .65 + .75 - .85$

$= \boxed{.55}$

4) Construct Venn Diagram



$P(A \text{ only}) = .65 - .55 = .1$

$P(B \text{ only}) = .75 - .55 = .2$

Total = 1

Use DeMorgan's Law to find

$P(\bar{A} \text{ and } \bar{B}) = P(\overline{A \text{ or } B}) = 1 - P(A \text{ or } B) = 1 - .85 = \boxed{.15}$

$P(\bar{A} \text{ or } \bar{B}) = P(\overline{A \text{ and } B}) = 1 - P(A \text{ and } B) = 1 - .55 = \boxed{.45}$

Jan 24-4:56 PM

Introduction to odds (Mathematical odds) SG 12

I flipped a coin 100 times, it landed 55 tails and 45 heads.

$P(\text{land tails}) = \frac{55}{100} = \frac{11}{20}$

$P(\text{land heads}) = \frac{45}{100} = \frac{9}{20}$

odds in favor of landing tails are

tails : # $\overline{\text{tails}}$

55 : 45

Divide by 5

$\boxed{11:9}$

odds against landing tails → $\boxed{9:11}$

Jan 24-5:10 PM

A standard deck of playing cards

52 cards & 4 Aces

odds in favor of drawing an Ace

Aces : # $\overline{\text{Aces}}$

4 : 48 → $\boxed{1:12}$

To reduce this 4 [:] 48 [Math] [1:▸Frac] [Enter]

odds against drawing an Ace → $\boxed{12:1}$

Jan 24-5:15 PM

Odds in favor of event E are $a : b$

" against " " " $b : a$

$$P(E) = \frac{a}{a+b}, \quad P(\overline{E}) = \frac{b}{a+b}$$

ex: odds in favor of LA Lakers win the championship this year are $3 : 37$.

3 → \$bet, 37 → \$Net

odds against $37 : 3$

$$P(W) = \frac{3}{3+37} = \frac{3}{40}$$

$$P(\overline{W}) = \frac{37}{3+37} = \frac{37}{40}$$

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If we have $P(E)$, then the odds in favor of event E are

$$P(E) : P(\overline{E})$$

Suppose $P(E) = .25$

1) $P(\overline{E}) = 1 - P(E) = \boxed{.75}$

2) odds in favor of event E

$$P(E) : P(\overline{E})$$

$$.25 : .75 \rightarrow \boxed{1 : 3}$$

3) odds against event E $\rightarrow \boxed{3 : 1}$

SG 12 → (Pages 1 & 2)

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Multiple-Actions

Flip a fair coin twice

T – Tails

H → Heads

T T
T H
H T
H H

A complete list of all possible outcomes

Sample Space

$P(2 \text{ tails}) = \frac{1}{4}$

$P(\text{exactly } 1 \text{ tail}) = \frac{2}{4} = \frac{1}{2}$

$P(\text{at least } 1 \text{ tail}) = \frac{3}{4}$

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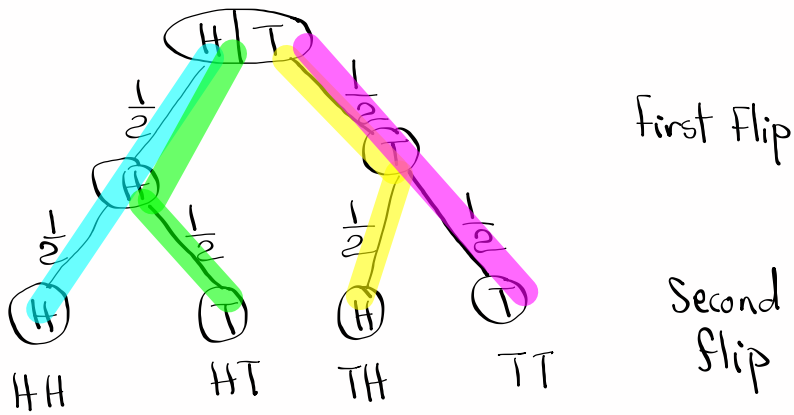
Let's look at Tree diagram



$P(2 \text{ Tails}) = \frac{1}{2} \cdot \frac{1}{2} = \boxed{\frac{1}{4}}$

$P(\text{exactly } 1 \text{ Tail}) = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \boxed{\frac{1}{2}}$

Jan 24-5:35 PM

Consider a standard deck of playing cards

52 Cards, 4 Aces

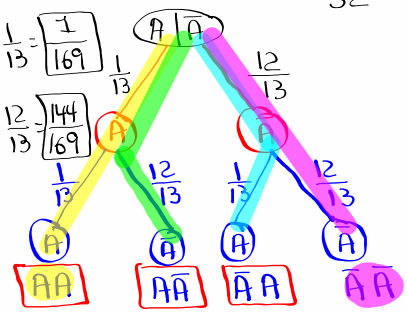
Draw 2 Cards $\boxed{\text{with replacement}}$

$P(\text{Draw Ace on first attempt}) = \frac{4}{52} = \frac{1}{13}$

$P(\text{Draw Ace on Second attempt}) = \frac{4}{52} = \frac{1}{13}$

$P(\text{2 Aces}) = \frac{1}{13} \cdot \frac{1}{13} = \boxed{\frac{1}{169}}$

$P(\text{No Aces}) = \frac{12}{13} \cdot \frac{12}{13} = \boxed{\frac{144}{169}}$



$P(\text{at least 1 ace}) = 1 - P(\text{No Aces}) = 1 - \frac{144}{169}$

$= \boxed{\frac{25}{169}}$

Jan 24-5:40 PM

Multiplication Rule

Keyword and

Multiple-Action event

$P(A \text{ and } B) = P(A) \cdot P(B|A)$

A happens,

then B happens

Given

Case I: independent events

One outcome does not change prob. of next one.

Flip a fair Coin $P(H) = \frac{1}{2}$, $P(T) = \frac{1}{2}$

New born babies $P(B) = \frac{1}{2}$, $P(G) = \frac{1}{2}$

Draw Cards with replacement $P(\text{Ace}) = \frac{1}{13}$ each time

Multiple choice questions with 4 choices but 1 Correct choice $P(\text{Correct}) = \frac{1}{4}$ on each question

$\boxed{\text{If A and B are independent events, } P(A \text{ and } B) = P(A) \cdot P(B)}$

Jan 24-6:04 PM

$P(A) = .6$, $P(B) = .3$

1) $P(\bar{A}) = 1 - P(A) = \boxed{.4}$

2) $P(\bar{B}) = 1 - P(B) = \boxed{.7}$

2) Find $P(A \text{ and } B)$ if they are $\boxed{\text{M.E.E.}}$

$\boxed{0}$

3) Find $P(A \text{ and } B)$ if they are $\boxed{\text{independent events.}}$

$P(A) \cdot P(B)$

$= (.6)(.3)$

$= \boxed{.18}$

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$P(\text{passing Stat class}) = .6$

Select 2 students, Assume they are independent events

P P

P $\bar{P}$

$\bar{P}$ P

$\bar{P}$ $\bar{P}$

$P(\text{both pass}) = (.6)(.6) = \boxed{.36}$

$P(\text{Neither one pass}) = (.4)(.4) = \boxed{.16}$

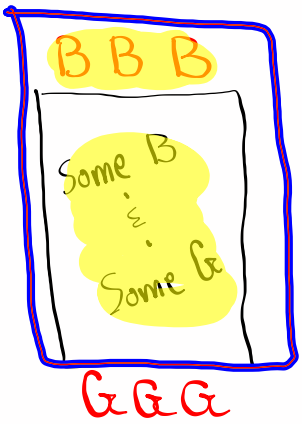
$P(1 \text{ pass}, 1 \overline{\text{pass}}) = (.6)(.4) + (.4)(.6) = \boxed{.48}$

Total = 1

Jan 24-6:17 PM

A family with 3 kids.

$P(B) = \frac{1}{2}$, $P(G) = \frac{1}{2}$ Independent events



$P(3 \text{ Boys}) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

P(at least 1 boy) =

$= 1 - P(GGG)$

Total Prob.

$= 1 - \frac{1}{8} = \boxed{\frac{7}{8}}$

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Suppose $P(A) = .5$, $P(B) = .4$, A & B are independent events

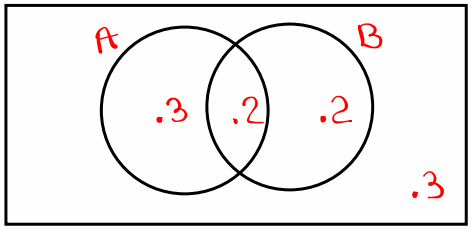
1) $P(\bar{A}) = 1 - P(A) = \boxed{.5}$

2) $P(\bar{B}) = 1 - P(B) = \boxed{.6}$

3) $P(A \text{ and } B) = P(A) \cdot P(B) = (.5)(.4) = \boxed{.2}$

4) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$= .5 + .4 - .2 = \boxed{.7}$



Total = 1

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A piggy bank has 4 dimes & 6 Nickels.

get 2 Coins with replacement.

N N → 10¢ $P(10¢) = \frac{6}{10} \cdot \frac{6}{10} = \frac{36}{100} = \boxed{.36}$

N D
D N → 15¢ $P(15¢) = \qquad = \boxed{.48}$

D D → 20¢ $P(20¢) = \frac{4}{10} \cdot \frac{4}{10} = \boxed{.16}$

L1	L2
10	.36
15	.48
20	.16

$\bar{x} = 14$

$S_x =$ blank

$n = 1$ ← Total Prob.

STAT → CALC

1: 1-Var Stats

List: L1

FreqList: L2

Calculate

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A Fundraiser sells 500 Tickets.

Each ticket is worth $10.

one ticket is randomly drawn,

winner gets a 80-in TV worth $1000.

Net gain (L1)	P(Net gain) (L2)	
10 − 1000	1/500	winning TKT
10 − 0	499/500	losing TKTS

STAT → CALC

1: 1-Var Stats

List: L1

FreqList: L2

Calculate

$\bar{x} = 8$ ← $8 profit Per TKT

$S_x =$ Blank

$n = 1$

Total Prob.

$\frac{\$4000 \text{ Total gain}}{500 \text{ Tickets}}$

$8/TKT

Jan 24-6:45 PM

A box has 5 Red & 15 Blue balls.

Select 3 balls $\boxed{\text{no replacement.}}$

Dependent Events

$$P(3\ \text{Red}) = \frac{\cancel{5}}{\cancel{20}} \cdot \frac{\cancel{4}}{\cancel{19}} \cdot \frac{\overset{1}{\cancel{3}}}{\underset{6}{\cancel{18}}} = \frac{1}{114}$$

$$P(3\ \text{Blue}) = \frac{15}{20} \cdot \frac{14}{19} \cdot \frac{13}{18} = \boxed{\frac{91}{228}}$$

Jan 24-6:53 PM